**1.** How many times is line (5) executed in the following pseudocode?

1. n = 10
2. m = 20
3. for *i* = 1 to *n* + 4
4. for *j* = 1 to *m*
5. print (*i*, *j*)

Construct a grid of values according to the pseudocode (optional)

Product Principle *mn*

In this problem, = **280**

**2.** How many times is line (3) executed in the following pseudocode? Express your answer as an efficient formula in terms of *n*.

1. for *i* = 1 to 5*n*
2. for *j* = 1 to *i*
3. print (*i*, *j*)

Make a grid of sample values

Make a set equation (|*S*|) using ellipses and Sum Principle

Add the first and last term together using Sum Principle (quickest way to add numbers is this)

Use the equation where:

*m* is the first number (so in this case would be 1)

*n* would be the 5*n* value instead

Multiply them and put them over 2

So, the **final formula** would be:

**3.** Determine the value of the output in the following pseudocode.

1. for *i* = 1 to 80
3. print *sum*

Make a sample set equation using

*i* for initial term (so in this case, 1)

*n* for ending term

Add the first and last term together using Sum Principle (quickest way to add numbers is this)

Use the equation where:

*m* is the first number (so in this case would be 1)

*n* would be the *n* value instead

Multiply them and put them over 2

Plug in 80 for *n*

The output is **3240**

**4.** Calculate each of the following. Your answer must be a number. No arithmetic operations are allowed in your answer. Please give 7 places after your decimal point if you use scientific notation.

1. 
2. 
3. 

Part A

Recognize that the form the factorial takes is

TI-84: 740 choose 50 (or cancel)

The answer is **1.749801543E78**.

Part B

Same as Part A

The answer you’d get is **7.628275985E15**.

Part C

Distribute into two fractions () using algebra

Simplify first expression to and cancel

Simplify second expression to and cancel

Subtract them

The answer is **446892**.

**5.** Simplify the expression:



Compare the factorials in the numerator and denominator

Expand the larger factorial such that it includes the smaller in the sequence

Cancel out the common factors between the numerator and denominator

Simplify further by multiplying or dividing the leftover expression

The answer is **16*n*4 - 16*n*3 - 4*n*2 + 4*n****.*

**6.** Calculate the following sum expressed in sigma notation:



Hit F3 to select the Calc menu

Hit the number 4 to pick the “sum” option

First, enter the expression in front of it (in this case, 5*k* + 8)

Then, comma variable *k*, comma the value *k* equals to (in this case, 1)

After that, comma the value at the top of the sigma (in this case, 14)

Close the parentheses (optional) and hit enter

Make sure to check the displayed sum is correct

The answer is **637**.

**7.** Find an efficient formula in terms of *m* and *n* for the following sum expressed in sigma notation. HINT: Use the handshake formula.



Make a sample sequence.

Add up the first and last number together.

Use the pair expression to get the number of pairs.

Multiply them together and divide by 2.

The **final formula**will be:

**8.** Calculate the following product expressed in pi notation:



Hit F3 to select the Calc menu

Hit the number 5 to pick the “product” option

First, enter the expression in front of it (in this case, 3*k* + 3)

Then, comma variable *k*, comma the value *k* equals to (in this case, 1)

After that, comma the value at the top of the sigma (in this case, 9)

Close the parentheses (optional) and hit enter

Make sure to check the displayed product is correct

The answer is **71425670400** (or in scientific notation, **7.14257E10**).

**9.** How many three-letter “words” can be made from 5 letters (FGHIJ) if repetition of letters

1. is allowed?
2. is not allowed?

Part A

If repetition is allowed, it’s multi-set. Order doesn’t matter.

= **125**

Part B

If repetition isn’t allowed, but order doesn’t matter, it’s a combination.

It doesn’t matter about the order of the letters, just that it doesn’t repeat.

= **60**

**10.** A president, a treasurer, and a secretary are to be chosen from a committee with 40 members. In how many ways could the three officers be chosen?

A combination for the same reasoning as above problem

Also remember, 3 spaces, possibilities decrease because an officer can’t hold two positions.

= **59280**

**11.** A bit is a 0 or a 1. A bit string of length 9 is a sequence of 9 digits, all of which are either 0 or 1.

1. How many bit strings of length 9 are there?
2. How many bit strings of length 9 or less are there? (Count the empty string of length 0 also.)

Part A

so 2 possibilities ^ 9 spaces = **512**

Part B

Create a sigma notation, where *k* is the number of spaces from 0 to 9.

Plug them into the calculator.



In this case, the sum would be **1023**.

**12.** Find how many positive integers with exactly four decimal digits, that is, positive integers between 1000 and 9999 inclusive, have the following properties:

1. are even
2. are divisible by 5 or by 7 (inclusive or)
3. are divisible by 7
4. are divisible by 5

Know there are 9000 numbers in the range.

Part A

Quotient Principle - *n* ways to do something, but for a specific way, *d* ways for it to happen (*n*/*d*)

= **4500**

Part B

Note the condition uses the inclusive “or”

Subtraction Rule - event can occur *m* or *n* ways (overlapping), number of ways is *m + n* minus the number of ways the event can commonly occur to the two different ways

To find out commonly occurrence,

product principle () = 35

quotient principle () = 257

*m* and *n* ways can be found in parts C and D

**2829**

Part C

Same reasoning as Part A

= **1286** (make sure to round down)

Part D

Same reasoning as Part A

= **1800**

**13.** 4 letter words are formed using the letters A, B, C, D, E, F, and G. How many such words are possible for each of the following conditions?

1. No condition is imposed
2. No letter can be repeated in a word
3. Each word must begin with the letter A
4. The letter C must be at the end
5. The second letter must be a vowel

Part A

where *n* is the number of letters there are, *k* is the number of spaces

74 = **2401**

Part B

This is also a combination (order doesn’t matter, repetition not allowed)

= **840**

Part C/D

Take careful note of the number of spaces (4)

First or last space taken by the value of A or C so there’s only 1 possibility for that

The rest of spaces are free to be whatever letter (7 possibilities each)

For both parts, **343**

Part E

Same as Part C/D, but there are 2 possibilities for that one space because there are 2 vowels

= **686**

**14.** A fair 6 sided die is rolled 4 times and the resulting sequence of 4 numbers is recorded.

1. How many different sequences are possible?
2. How many different sequences consist entirely of even numbers?
3. How many different sequences are possible if the first, third, and fourth numbers must be the same?

Part A

, where *n* is the number of possibilities and *k* is the number of spaces

= **1296**

Part B

Same as Part A, but reduced number of possibilities due to condition for even numbers

= **81**

Part C

If 1st, 3rd, 4th positions fixed same, 1 free position that can be any numbers

where *n* is the number of possibilities, *k* is the number of positions free to be any number

= **36**

**15.** A DNA sequence can be represented as the string of the letters ACTG (short for adenine, cytosine, guanine, and thymine).

1. How many DNA sequences are exactly 23 letters long?
2. Given a DNA sequence of length 23, how many single letter mutations are possible?
3. Given a DNA sequence of length 23, how many double letter mutations are possible?

Part A

= = **70368744177664** or **7.036874418E13** in scientific notation.

Part B

Combination (*n* choose *k*) where *n* is total group (length) and *k* is the number of items chosen (single letter mutations = 1)

23 choose 1 = 23

Multiply by 3 becuz 3 possible mutations for 3 other letters it could mutate to

= **69**

Part C

Same thing as Part B but

23 choose 2 = 253

Multiply by 9 becuz double letter mutations (if 2 spaces mutate, 3 possibilities each, so for product)

= **2277**

**16.** How many different one-to-one functions can be defined that map the domain to the range ?

One-to-one means each input must have an output, output not shared (no repetition of output)

14 options (outputs to choose from) for 7 inputs

14 outputs choose 7 inputs

Combination because repetition of output is not allowed and order of outputs doesn’t matter

What matters is that each input has an output

Equals **17297280** or **1.72973E7** in scientific notation

**17.** How many different one-to-one functions can be defined that map the domain to the range such that *f* is NOT one-to-one?

Find the total number of functions () so = = 170859375

Find the number of one-to-one functions (combination) so 15 choose 7 = 32432400 (like #16)

Subtract them both **138426975** or **1.38427E8** in scientific notation

**18.** How many different one-to-one functions can be defined that map the domain to the range ?

Since each input can be paired with an output (no outputs leftover), 7!

Because same number of values in the domain and range

**5040**

**19.** Count the number *X* of functions so that φ is not onto: .

Onto means every output needs to be hit

Find the total number of functions () so = 3125

Find the number of onto functions (combination) so 5! = 120

Because same number of values in domain and range

Subtract them to get **3005**

**20.** Suppose that 671 tennis players want to play an elimination tournament. That means: they pair up, at random, for each round; if the number of players before the round begins is odd, one of them, chosen at random, sits out that round. The winners of each round, and the odd one who sat it out (if there was an odd one), play in the next round, till, finally, there is only one winner, the champion. What is the total number of matches to be played all together, in all the rounds of the tournament?



Replace the *n* with 671 and solve to get **670**